Automated inference of rules with exception from past legal cases using ASP: Supplementary material

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Abstract

Supplementary document for our LPNMR 2015 submission containing proofs and full example that were not included in the submission due to space limitation.

1 Learning Example in Legal Reasoning

This section contains the all the steps in the computation of the judgement theory from our legal example.

Firstly, all the given past cases are expressed, using our meta-level representation, by the following set of facts are in $meta(CB)$:

```
cb_id(c0).       factor(ria).              in_rule(c3,neg_dwe,ooo).
cb_id(c1).       factor(neg_dot).          in_rule(c3,neg_dwe,rpl).
cb_id(c2).       default_head(neg_dwe).    in_rule(c3,neg_dwe,far).
cb_id(c3).       default_id(c0).           in_rule(c3,neg_dwe,ria).
cb_id(c4).       is_rule(c0,neg_dwe).      is_rule(c4,dwe).
cb_id(c5).       is_rule(c1,neg_dwe).      in_rule(c4,dwe,neg_dot).
factor(dot).     in_rule(c1,neg_dwe,dot).  in_rule(c4,dwe,fad).
factor(fad).     is_rule(c2,dwe).          is_rule(c5,neg_dwe).
factor(dia).     in_rule(c2,dwe,dot).      in_rule(c5,neg_dwe,neg_dot).
factor(ooo).     in_rule(c2,dwe,ooo).      in_rule(c5,neg_dwe,fad).
factor(rpl).     is_rule(c3,neg_dwe).      in_rule(c5,neg_dwe,dia).
factor(far).     in_rule(c3,neg_dwe,dot).
```

The optimal answer set A_{opt} of Π_{CB} contains the following raw attacks, and relevant attacks:

```
raw_attack(c2,c0).     raw_attack(c3,c2).     attack(c4,c0).
raw_attack(c4,c0).     raw_attack(c5,c4).     attack(c3,c2).
raw_attack(c2,c1).     attack(c2,c0).         attack(c5,c4).
```

Note that $c2 \to_r c1$ is not a relevant attack as it violates the second condition of Definition 3 as $c2 \to_r c0$. Using $meta(CB)$ and the relevant attacks, the following arguments are deduced:

```
argument(c2,c0,ooo).       argument(c4,c0,fad).   argument(c3,c2,rpl).
argument(c2,c0,dot).       argument(c3,c2,ria).   argument(c5,c4,dia).
argument(c4,c0,neg_dot).   argument(c3,c2,far).
```

These are used with Π_{gen} to generate the answer set containing the following meta-level representation of the judgement theory:

```
is_rule(r0,neg_dwe).
in_rule(r0,neg_dwe,not_ab0).
in_rule(r0,neg_dwe,not_ab1).
is_rule(r4,ab0).
in_rule(r4,ab0,dot).
in_rule(r4,ab0,ooo).
in_rule(r4,ab0,not_ab2).
is_rule(r5,ab1).
in_rule(r5,ab1,neg_dot).
in_rule(r5,ab1,fad).
in_rule(r5,ab1,not_ab3).
is_rule(r2,ab2).
in_rule(r2,ab2,rpl).
in_rule(r2,ab2,far).
in_rule(r2,ab2,ria).
is_rule(r3,ab3).
in_rule(r3,ab3,dia).
```

This corresponds to the program:

```
neg_dwe :- not ab0, not ab1.
ab0 :- dot, ooo, not ab2.
ab1 :- fad, neg_dot, not ab3.
ab2 :- far, ria, rpl.
ab3 :- dia.
```

2 Correctness of the Generated Program

This section contains proofs for the lemmas used to prove the proposition in the submission. For completeness, definitions and programs from the submission are also included.

2.1 Definitions

Definition 1 (Casebase)**.** *Let F be a set of elements called* factors*. A* case *is a subset of F. A* case with judgement *is a pair $cj = \langle c, j \rangle$, where c is a case and $j \in \{+, -\}$. The set c is also referred to as the set of factors included in a case with judgement. Given a case with judgement cj, $case(cj)$ denotes the set of factors included in cj and $judgement(cj) = j$ denotes the judgement decision taken in the case. A* casebase*, denoted with CB, is a set of cases with judgements, namely a subset of $\mathcal{P}(F) \times \{+, -\}$, where $\mathcal{P}(F)$ is the powerset of F.*

Definition 2 (Raw Attack)**.** *Let CB be a casebase. The* raw attack *relation is a set $RA \subseteq CB \times CB$ defined as the set of all pairs $\langle cj_1, cj_2 \rangle$ such that $\langle cj_1, cj_2 \rangle \in RA$ if and only if $case(cj_1) \supset case(cj_2)$ and $judgement(cj_1) \neq judgement(cj_2)$. For every pair $\langle cj_1, cj_2 \rangle \in RA$, we say cj_1* raw attacks *cj_2 and we write $cj_1 \rightarrow_r cj_2$.*

Definition 3 (Relevant Attack)**.** *Let CB be a casebase. The* relevant attack *relation $AT \subseteq RA$ is the set of pairs $\langle cj_1, cj_2 \rangle \in RA$ such that:*

- *$\langle cj_1, cj_2 \rangle \in AT$ if $case(cj_2) = \emptyset$ and there is no $cj_3 \rightarrow_r cj_2$ in RA such that $case(cj_1) \supset case(cj_3)$*
- *$\langle cj_1, cj_2 \rangle \in AT$ if there exists $\langle cj_2, cj_4 \rangle \in AT$ and there is no $cj_5 \rightarrow_r cj_2$ in RA such that $case(cj_1) \supset case(cj_5)$*
- *nothing else is in AT.*

Each element $\langle cj_1, cj_2 \rangle \in AT$ is denoted as $cj_1 \rightarrow cj_2$.

Definition 4 (Argument)**.** *Let CB be a casebase and let AT be the set of relevant attacks with respect to CB. For each pair $\langle cj_1, cj_2 \rangle \in AT$, the set of factors representing the attack from cj_1 to cj_2 is given by $\alpha(cj_1, cj_2) = case(cj_1) - case(cj_2)$.*

Definition 5 (Active Case with Judgement)**.** *Let CB be a casebase, AT be its corresponding set of relevant attacks, and c be a case. A case with judgement $cj \in CB$ is active with respect to c if and only if $case(cj) \subseteq c$, and for all $\langle cj_n, cj \rangle \in AT$, either $case(cj_n) \nsubseteq c$ or cj_n is not active with respect to c.*

Definition 6 (Predicted judgement)**.** *Let CB be a casebase, AT be the set of relevant attacks with respect to CB, and c be an unseen case (for all $cj \in CB$, $case(cj) \neq c$). The unique predicted judgement of c, denoted with $pj(c)$, is equal to the default judgement j_0 if and only if $\langle \emptyset, j_0 \rangle$ is active with respect to c.*

Definition 7 (Rule representation of a case with judgement)**.** *Let CB be a casebase. Each $cj \in CB$ can be expressed as a definite clause $r(cj)$ called a* rule*: $judgement(cj) :- f_1, \ldots, f_n$. where $f_i \in case(cj)$, for $1 \leq i \leq n$.*

Definition 8 (Casebase meta-level representation)**.** *Let CB be a casebase. Its meta-level representation, $meta(CB)$, is defined as:*

$$meta(CB) = \bigcup_{cj \in CB} \mu(r(cj)) \cup \tau(CB) \cup \delta(CB)$$

$\tau(CB) = \{\mathtt{factor(f_i)} | \mathtt{f_i} \in \mathtt{F}\}$, $\delta(CB)$ is the meta-information about the default case, $\delta(CB) = \{\mathtt{default\_id(id(r(cj_0)))}, \mathtt{default\_head(judgement(cj_0))}\}$, and the function μ is defined as follows:

$$\mu(r(cj)) = \begin{cases} \mathtt{cb\_id(id(r(cj)))}. & \\ \mathtt{is\_rule(id(r(cj)), judgement(cj))}. & \\ \mathtt{in\_rule(id(r(cj)), judgement(cj), f_i)}. & \text{for each } f_i \in case(cj) \end{cases}$$

Definition 9. *Given a casebase CB and its relevant attacks AT, the judgement theory T is the set of rules such that given a new case c, T derives the default judgement j_0 if and only if $pj(c) = j_0$. Let $ab(cj_i \rightarrow cj_j)$ be the reified atom of $cj_i \rightarrow cj_j$ in AT. The following rules are in the judgement theory*[1]

- *For empty case with judgement cj_0, and $cj_1 \rightarrow cj_0, \ldots, cj_n \rightarrow cj_0 \in AT$*
  $judgement(cj_0) : -not\ ab(cj_1 \rightarrow cj_0), \ldots, not\ ab(cj_n \rightarrow cj_0)$.
- *For $f_1, \ldots, f_m \in \alpha(cj_x, cj_y)$, and $cj_{x+1} \rightarrow cj_x, \ldots, cj_{x+k} \rightarrow cj_x \in AT$*
  $ab(cj_x \rightarrow cj_y) : -f_1, \ldots, f_m, not\ ab(cj_{x+1} \rightarrow cj_x), \ldots, not\ ab(cj_{x+k} \rightarrow cj_x)$.

2.2 Programs

$$\Pi_{raw} = \begin{cases} \mathtt{raw\_attack(ID_1, ID_2) : -factor\_subset(ID_2, ID_1), is\_rule(ID_1, H_1),} \\ \quad \mathtt{is\_rule(ID_2, H_2), H_1 \neq H_2.} \\ \mathtt{factor\_subset(ID_1, ID_2) : -cb\_id(ID_1), cb\_id(ID_2),} \\ \quad \mathtt{not\ not\_factor\_subset(ID_1, ID_2).} \\ \mathtt{not\_factor\_subset(ID_1, ID_2) : -cb\_id(ID_1), cb\_id(ID_2), factor(B),} \\ \quad \mathtt{in\_rule(ID_1, H_1, B), is\_rule(ID_2, H_2), not\ in\_rule(ID_2, H_2, B).} \end{cases}$$

$$\Pi_{rev_1} = \begin{cases} \mathtt{0\ \{attack(ID_1, ID_2)\}\ 1 : -raw\_attack(ID_1, ID_2).} \\ \mathtt{: -attack(ID_1, ID_2), not\ attackee(ID_2).} \\ \mathtt{: -attack(ID_1, ID_2), raw\_attack(ID_3, ID_2), factor\_subset(ID_3, ID_1).} \\ \mathtt{attackee(ID) : -default\_id(ID).} \\ \mathtt{attackee(ID_2) : -attack(ID_2, ID_4).} \end{cases}$$

[1]where $\alpha/2$ represents arguments as described in Definition 4

$\Pi_{rev_2} = \{\#\texttt{maximise}\{\texttt{attack}(\texttt{ID}_1, \texttt{ID}_2)\}.\}$

$$\Pi_{arg} = \begin{cases} \texttt{argument}(\texttt{ID}_1, \texttt{ID}_2, \texttt{Arg}) : -\texttt{attack}(\texttt{ID}_1, \texttt{ID}_2), \texttt{in\_rule}(\texttt{ID}_1, \texttt{H}_1, \texttt{Arg}), \\ \qquad \texttt{is\_rule}(\texttt{ID}_2, \texttt{H}_2), \texttt{not in\_rule}(\texttt{ID}_2, \texttt{H}_2, \texttt{Arg}). \end{cases}$$

$$\Pi_{gen_1} = \begin{cases} 1\ \{\texttt{id\_attack\_link}(\texttt{AID}, \texttt{a}(\texttt{ID}_1, \texttt{ID}_2)) : \texttt{abnormal}(\texttt{AID}, \texttt{Ab})\}\ 1 \\ \quad : -\texttt{attack}(\texttt{ID}_1, \texttt{ID}_2). \\ : -\texttt{id\_attack\_link}(\texttt{AID}_1, \texttt{At}), \texttt{id\_attack\_link}(\texttt{AID}_2, \texttt{At}), \texttt{AID}_1 \neq \texttt{AID}_2. \\ : -\texttt{id\_attack\_link}(\texttt{AID}, \texttt{At}_1), \texttt{id\_attack\_link}(\texttt{AID}, \texttt{At}_2), \texttt{At}_1 \neq \texttt{At}_2. \end{cases}$$

$$\Pi_{gen_2} = \begin{cases} \texttt{gen\_id}(\texttt{r0}). \\ \texttt{gen\_id}(\texttt{ri}). \quad \texttt{abnormal}(\texttt{ri}, \texttt{abi}). \quad \texttt{negated\_abnormal}(\texttt{ri}, \texttt{not\_abi}). \\ \quad \textit{For } 1 \leq i \leq n, \textit{where } n \textit{ is the number of relevant attacks} \end{cases}$$

$$\Pi_{gen_3} = \begin{cases} \texttt{is\_rule}(\texttt{r0}, \texttt{Def}) : -\texttt{default\_head}(\texttt{Def}). \\ \texttt{in\_rule}(\texttt{r0}, \texttt{Def}, \texttt{NAb}) : -\texttt{default\_head}(\texttt{Def}), \texttt{default\_id}(\texttt{ID2}), \\ \quad \texttt{id\_attack\_link}(\texttt{AID}, \texttt{a}(\texttt{ID}_1, \texttt{ID}_2)), \texttt{negated\_abnormal}(\texttt{AID}, \texttt{NAb}). \end{cases}$$

$$\Pi_{gen_4} = \begin{cases} \texttt{is\_rule}(\texttt{AID}, \texttt{Ab}) : -\texttt{id\_attack\_link}(\texttt{AID}, \texttt{At}), \texttt{abnormal}(\texttt{AID}, \texttt{Ab}). \\ \texttt{in\_rule}(\texttt{AID}, \texttt{H}, \texttt{Arg}) : -\texttt{is\_rule}(\texttt{AID}, \texttt{H}), \texttt{argument}(\texttt{ID1}, \texttt{ID2}, \texttt{Arg}). \\ \quad \texttt{id\_attack\_link}(\texttt{AID}, \texttt{a}(\texttt{ID}_1, \texttt{ID}_2)), \\ \texttt{in\_rule}(\texttt{AID}_1, \texttt{H}, \texttt{NAb}) : -\texttt{is\_rule}(\texttt{AID}_1, \texttt{H}), \texttt{negated\_abnormal}(\texttt{AID}_2, \texttt{NAb}), \\ \quad \texttt{id\_attack\_link}(\texttt{AID}_1, \texttt{a}(\texttt{ID}_2, \texttt{ID}_1)), \texttt{id\_attack\_link}(\texttt{AID}_2, \texttt{a}(\texttt{ID}_3, \texttt{ID}_2)). \end{cases}$$

$\Pi_{rev} = \Pi_{rev_1} \cup \Pi_{rev_2}$

$\Pi_{CB} = meta(CB) \cup \Pi_{raw} \cup \Pi_{rev}$.

$\Pi_{gen} = \Pi_{gen_1} \cup \Pi_{gen_2} \cup \Pi_{gen_3} \cup \Pi_{gen_4}$

$\Pi_{JT} = \Pi_{gen} \cup AR \cup \{default\_head(judgement(cj_0)), default\_id(id(r(cj_0)))\}$ where AR is the answer set of arguments of the relevant attacks computed using Π_{arg}.

2.3 Proofs

Definition 10. *Let $\{S\} = AS(meta(CB) \cup \Pi_{raw})$, CB be a casebase, AT its relevant attacks, and $Q = \{attack(id(r(cj_i)), id(r(cj_j))), attackee(id(r(cj_j))) \mid \langle cj_i, cj_j \rangle \in AT\}$. $ans(CB, AT)$ is the interpretation $S \cup Q \cup \{attackee(id(r(cj_0)))\}$.*

Lemma 1. *Let S be the unique answer set of $meta(CB) \cup \Pi_{raw}$ and let $\pi_p(A)$ denote the projection of A over p. A set $A \in AS(\Pi_{CB})$ iff:*

(i) $\pi_{attack}(A) \subseteq \pi_{raw\_attack}(A)$

(ii) For all cj_1 and cj_2, if $attack(id(r(cj_1)), id(r(cj_2))) \in A$ then $attackee(id(r(cj_2))) \in A$

(iii) For all cj_1 and cj_2, if $attack(id(r(cj_1)), id(r(cj_2))) \in A$ then there does not exists cj_3 such that $raw\_attack(id(r(cj_3)), id(r(cj_2))) \in A$ and $factor\_subset(id(r(cj_3)), id(r(cj_1))) \in A$)

(iv) For all cj, $attackee(id(r(cj)) \in A$ if and only if $default\_id(id(r(cj_3))$ is true or $attack(id(r(cj)), id(r(cj_x))) \in A$

(v) Let $\mathcal{L}$ be the language of $meta(CB) \cup \Pi_{raw}$, then for all s, $s \in \pi_{\mathcal{L}}(A)$ if and only if $s \in S$

Proof. The program Π_{CB} can be split into two programs $meta(CB) \cup \Pi_{raw}$ and Π_{rev} such that the answer set S of $meta(CB) \cup \Pi_{raw}$ forms the inputs to Π_{rev}. This simplifies Π_{CB} into facts representing the casebase $meta(CB)$, raw attacks RA, and sub factors and non sub factors between cases of the casebase SF together with the program Π_{rev}. We now show that $X \in AS(\Pi_{CB})$ iff X satisfies (i)-(v). Firstly, assume X satisfies (i)-(v). The reduct of Π_{CB}, using simplified reduct [1], wrt X is:

1. $\mathtt{attack(id_1, id_2) :- raw\_attack(id_1, id_2)}$. For all $attack(id_1, id_2) \in X$ and $raw\_attack(id_1, id_2) \in S$)
2. $\bot \mathtt{:- attack(id_1, id_2)}$. For all $attackee(id_2) \notin X$
3. $\mathtt{:- attack(id_1, id_2), raw\_attack(id_3, id_2), factor\_subset(id_3, id_1)}$. For all $raw\_attack(id_3, id_2) \in X$ and $factor\_subset(id_3, id_1) \in X$
4. $\mathtt{attackee(id) :- default\_id(id)}$. For all $default\_id(id) \in X$)
5. $\mathtt{attackee(id_2) :- attack(id_2, id_4)}$. For all $raw\_attack(id_2, id_4) \in X$
6. $\mathtt{s}$. For each $s \in S$

Consider the reduct without the constraints (line 2-3), since it is stratified, it has a unique minimal model M. Constructing M (using the iterated fixpoint operator) yields X. Thus X is the minimal model of the reduct without constraints. Now, if we consider the constraints, the first (line 2) cannot be satisfied due to property (ii), and the second (line 3) cannot be satisfied due to property (iii). Thus X is the minimal model of the reduct, and hence, an answer set.

Next assume that $X \in AS(\Pi_{CB})$. We show that X satisfies (i)-(v).

i Assume X violates (i). Then there exists $attack(id_1, id_2) \in X$ st $raw\_attack(id_1, id_2) \notin X$. This contradicts the sole definition of $attack/2$ in the reduct.

ii Assume X violates (ii). Then there exists $attack(id_1, id_2) \in X$ but $attackee(id_2) \notin X$. This violates a constraint of Π_{rev}. Contradiction, as $X \in AS(\Pi_{CB})$.

iii Similarly if X violates (iii) then X also violates a constraint of Π_{rev}.

iv The default id is reserved for the empty case, thus lines 4 and 5 of the reduct, $attackee(id) \in X$ iff id is the id of the empty case, or $attack(id, id') \in X$.

v If X violates (v), then either there exists $s \in \Pi_{\mathcal{L}}(X)$ st $s \notin S$, or vice versa; however, this cannot occur, since all elements of S are facts in the reduct.

□

Lemma 2. *Given two answer sets $A_1, A_2 \in AS(\Pi_{CB})$, then $A_1 \cup A_2 \in AS(\Pi_{CB})$.*

Proof. By lemma 1 A_1 and A_2 both satisfy (i)-(v). Informally, (i) and (ii) are satisfied by $A_1 \cup A_2$ as they were by both A_1 and A_2. (iii)-(v) are satisfied as the raw_attack's, $factor\_subset$'s and $default\_id$'s are the same in A_1, A_2 and therefore $A_1 \cup A_2$ (so if A_1 and A_2 satisfy (iii)-(v), then so does $A_1 \cup A_2$). □

Lemma 3. *Given a casebase* CB*, its raw attacks* RA*, and its relevant attack* AT*,* $ans(CB, AT) \in AS(\Pi_{CB})$*.*

Proof. We show that $ans(CB, AT)$ satisfies (i)-(v). Property (i) is given by Definition 3, property (v) is given from the definition of $ans(CB, AT)$, and we can show inductively that property (ii)-(iv) holds:

- **Base case** ($AT = \{\langle cj_1, cj_2\rangle \mid case(cj_2) = \emptyset$ and there is no $(cj_3 \rightarrow_r cj_2) \in RA$ st $case(cj_1) \supset case(cj_3)\}$):
  - *(ii)* $attackee(id(r(cj_2))) \in ans(CB, AT)$ by Definition 10 as $cj_2 = \emptyset$. Hence *(ii)* is satisfied.
  - *(iii)* For each $attack(id(r(cj_1)), id(r(cj_2))) \in ans(CB, AT)$, $\langle cj_1, cj_2\rangle \in AT$. Hence there is no $(cj_3 \rightarrow_r cj_2) \in RA$ st $case(cj_1) \supset case(cj_3)$. So by definition 11, there is no cj_3 st $raw\_attack(r(id(cj_3)), r(id(cj_2))) \in ans(CB, AT)$ and $factor\_subset(r(id(cj_3)), r(id(cj_1))) \in ans(CB, AT)$.
  - *(iv)* This follows immediately from definition 11.
- **Inductive hypothesis**: $ans(CB, AT')$ satisfies *(ii)-(iv)*.
- **Inductive step** ($\langle cj_1, cj_2\rangle \in RA$ st $AT = AT' \cup \{\langle cj_1, cj_2\rangle\}$ and the is some $\langle cj_2, cj_4\rangle \in AT'$ st there is no $(cj_5 \rightarrow_r cj_2) \in RA$ for which $case(cj_1) \supset case(cj_5)$):

  Note that by Definition 10, $ans(CB, AT') \subseteq ans(CB, AT)$.
  - *(ii)* By the inductive hypothesis, $attackee(id(r(cj_j))) \in ans(CB, AT')$ for each $attack(r(id(cj_i)), r(id(cj_j))) \in ans(CB, AT')$. As there is some $\langle cj_2, cj_4\rangle \in AT'$, $attackee(id(r(cj_2))) \in ans(CB, AT)$ by Definition 10.
  - *(iii)* By the inductive hypothesis, $ans(CB, AT')$ satisfies (iii), so it remains to show that there is no $raw\_attack(r(id(cj_3)), r(id(cj_2))) \in ans(CB, AT)$ such that $factor\_subset(r(id(cj_3)), r(id(cj_1))) \in ans(CB, AT)$. But this holds as there is no $(cj_5 \rightarrow_r cj_2) \in RA$ for which $case(cj_1) \supset case(cj_5)$.
  - *(iv)* By the inductive hypothesis, $ans(CB, AT')$ satisfies (iv), so as $\langle cj_1, cj_2\rangle$ adds $attackee(cj_2)$ (by Definition 10), $ans(CB, AT)$ satisfies (iv).

Hence, by lemma 1, $ans(CB, AT)$ is an answer set of Π_{CB}. □

Proposition 1. *Given a casebase* CB *with relevant attacks* AT*,* $ans(CB, AT)$ *is the unique optimal answer set of* Π_{CB}*.*

Proof. By Lemma 3, $ans(CB, AT) \in AS(\Pi_{CB})$. Assume for contradiction there exists $A'_{opt} \in AS(\Pi_{CB})$ such that $A'_{opt} \neq ans(CB, AT)$ and A'_{opt} is not less optimal than $ans(CB, AT)$. Both A'_{opt} and $ans(CB, AT)$ satisfy (v) and $attackee/1$ is defined using $attack/2$, so there must be an attack instance in A'_{opt} which is not in $ans(CB, AT)$. Let $D = A'_{opt} \setminus ans(CB, AT)$. By Lemma 2 $A'_{opt} \cup ans(CB, AT)$ must also be an answer set of Π_{CB}, and $A'_{opt} \cup ans(CB, AT) = D \cup ans(CB, AT)$. Let d be an $attack/2$ instance such that $attack(id_1, id_2) \in D$ and for all id_3 such that $attack(id_2, id_3) \notin D$. Let $A' = \{d\} \cup \{attackee(id_1)\} \cup$

$ans(CB, AT)$. A' satisfies (i), (iii), and (v) as they are also satisfied by $D \cup ans(CB, AT)$. As $d \in A'_{opt}$ either $default(id_2) \in A'_{opt}$ or $attack(id_2, id_3) \in A'_{opt}$. In the first case $default(id_2) \in ans(CB, AT \cup \{d\})$ and in the second since $attack(id_2, id_3) \in A'_{opt}$ but $attack(id_2, id_3) \notin D$ it must be that $attack(id_2, id_3) \in A'_{opt} \cap ans(CB, AT)$ and hence $attack(id_2, id_3) \in A'$. So in both cases A' satisfies (ii). Since $ans(CB, AT)$ satisfies (iv), so would A'. This shows that A' is also an answer set of Π_{CB}. However, let $\langle cj_1, cj_2 \rangle$ be the relevant attack corresponding to d, then by (ii), (iii) and (iv) and Definition 3 then $\langle cj_1, cj_2 \rangle \in AT$ must be true and so $d \in ans(CB, AT)$. Contradiction as $d \in A'_{opt} \setminus ans(CB, AT)$. Thus $ans(CB, AT)$ is the unique optimal answer set of Π_{CB}. □

Lemma 4. *Given a casebase CB and associated judgement theory T. Then for $A \in AS(\Pi_{JT})$, $\pi_{in\_rule, is\_rule}(A) = \mu(T)$.*

Π_{gen} generates rules from the default judgement where the body literals are either factors of the casebase, or negated abnormalities, which is the same structure as the judgement theory. Apart from the Skolemisation Π_{gen} is a definite program, thus while there are multiple answer sets to Π_{gen} due to the Skolemisation, they are all equivalent with respect to the renaming of abnormalities. The meta-level information generated by Π_{gen} directly corresponds to the structure of the judgement theory in Definition 9, linking the default judgement with abnormalities linked with attacks against the empty case with judgement, and linking subsequent abnormalities with other normalities representing attacks against it. Thus, provided that AT (and consequently AR) is correct with respect to CB, then the generated meta-level representation is the encoding of T.

Lemma 5. *Given a casebase CB with associated judgement theory T, let c be a new case, given as a set of factors. From Definition 9 for all abnormality rules ab in T there exists a sequence of rules not $head(ab) \in body(ab_{x_1})$, ..., not $head(ab_{x_n}) \in body(def)$ in T, where $n \geq 0$. The union of all its factors corresponds to a $cj \in CB$, and $cj \to cj_y$ for some $cj_y \in CB$. Abnormalities with a sequence such that $case(cj) \subseteq c$ is denoted by $seq(ab)$.*

Lemma 6. *Given a casebase CB with associated judgement theory T, and a new case c. Let $T_c = \{def\} \cup \{ab | ab \in T, seq(ab)\}$, and $T_{ex} = T \setminus T_c$. Then T derives j_0 iff T_c derives j_0*

Proof. The program $T_{ex} \cup c$ does not effect the derivation of j_0. For a rule ab to be in T_{ex}, $seq(ab)$ does not hold indicating that either there exists $f \in fs(ab)$ but $f \notin c$ in which case *not ab* holds, or there exists another abnormality closer to def containing factors not in c, making ab irrelevant to the inference of j_0. □

Proposition 2. *Given a casebase CB with associated judgement theory T, and a new case c. Let $\{A_T\} = AS(T \cup c)$. Then $j_0 \in A_T$ if and only if $pj(c) = j_0$.*

Proof. Let A_c be the answer set of T_c. By Lemma 6 we can reduce the problem to $j_0 \in A_c$ if and only if $pj(c) = j_0$. Let $dep(r)$ be the number of abnormalities n a rule $r \in T_c$ depends on (the number of abnormalities in T_c that are in its body and bodies of linked sub-rules). We can show that for all $r \in T_c$, $head(r) \in A_c$ iff all abnormalities in $body(r)$ are not in A_c. We use strong induction on $dep(r)$. Assume as inductive hypothesis that for all $r' \in T_c$ such that $dep(r') < dep(r)$,

$head(r') \in A_c$ iff all abnormalities in $body(r')$ are not in A_c. There are two cases:

- **Case 1:** $dep(r) = 0$. As $r \in T_c$ then all $f \in fs(ab)$ must be in c. If there is any *not* $ab \in body(r)$ then they must be in T_{ex}, thus $head(r) \in A_c$.
- **Case 2:** $dep(r) > 0$. All $f \in fs(r)$ are in c. For all abnormalities ab in $body(r)$, $dep(ab) < dep(r)$. By the inductive hypothesis each $head(ab) \in A_c$ iff all abnormalities in its body are not in A_c. Thus, $head(r) \in A_c$ iff all $head(ab)$ not in A_c.

Thus $j_0 \in A_c$ iff all of its abnormalities are not in A_c. In such situation all cases with judgement attacking the the empty case with judgement are not active with respect to c and by Definition 5 $pj(c) = j_0$. □

References

[1] Mark Law, Alessandra Russo, and Krysia Broda. Simplified reduct for choice rules in asp. Technical Report DTR2015-2, Imperial College of Science, Technology and Medicine, Department of Computing, 2015.