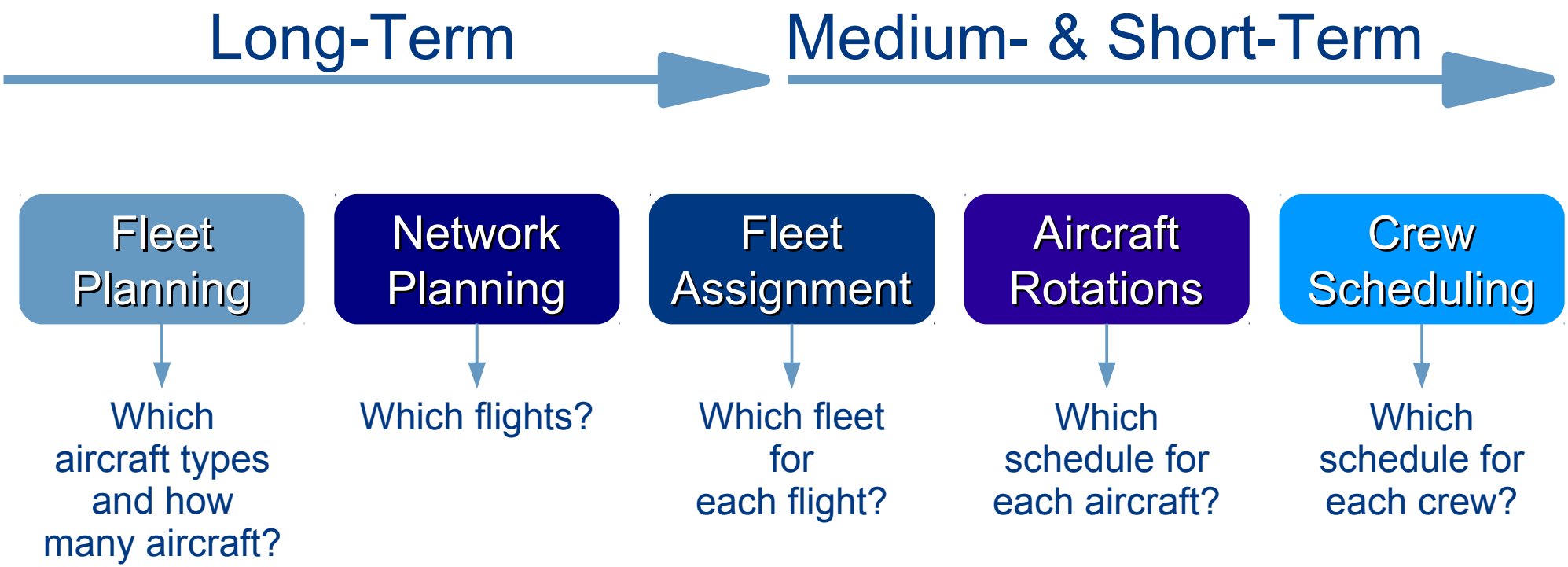
decisal
making decisions optimal

Airline planning & airline scheduling with Unified Optimization

SchedulAir

Overview

- Unified Optimization
- Benders decomposition
- decisal


Long-Term
Medium- & Short-Term
Fleet Planning
Network Planning
Fleet Assignment
Aircraft Rotations
Crew Scheduling
Which aircraft types and how many aircraft?
Which flights?
Which fleet for each flight?
Which schedule for each aircraft?
Which schedule for each crew?

SchedulAir unified management & optimization

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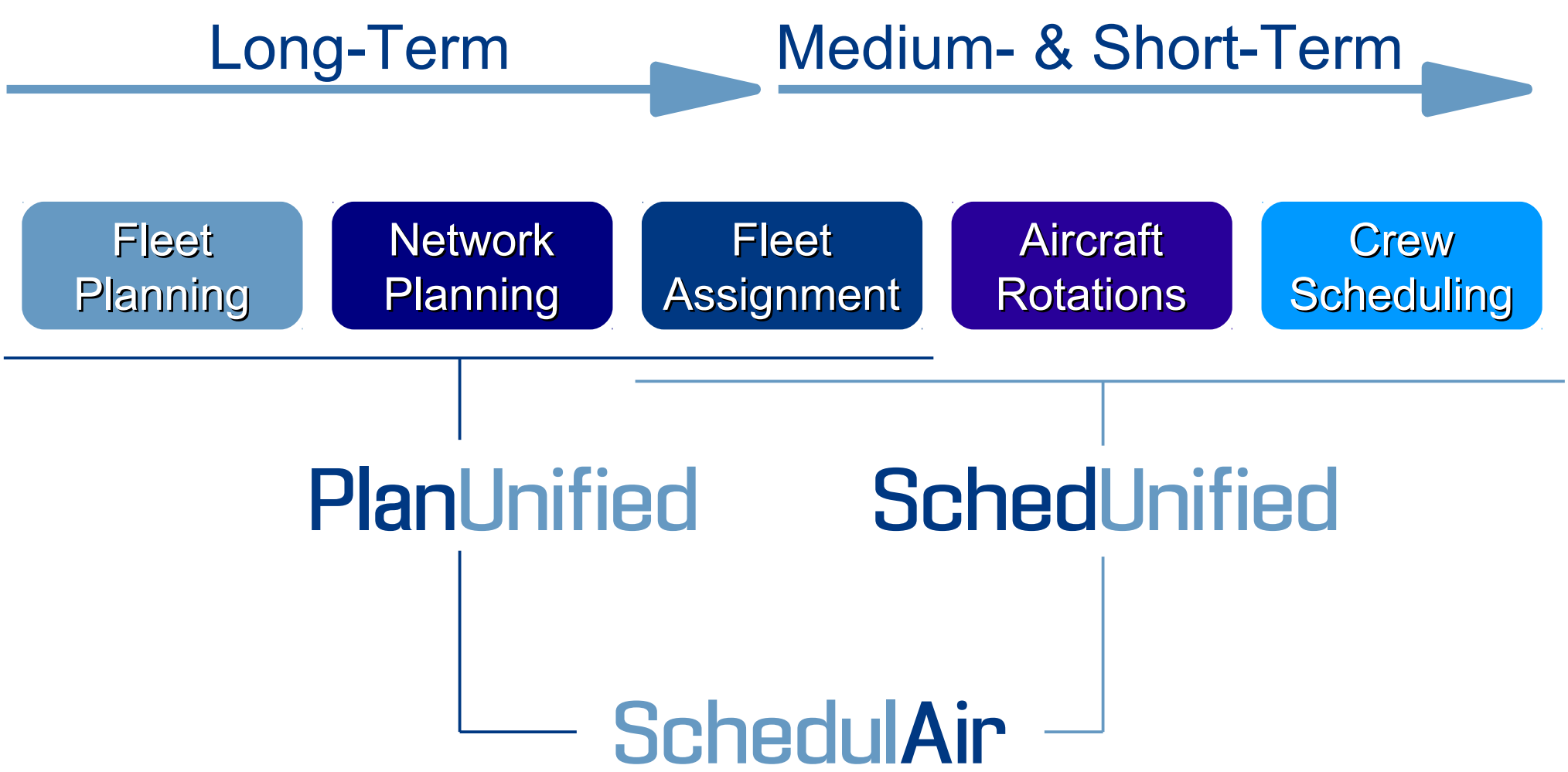
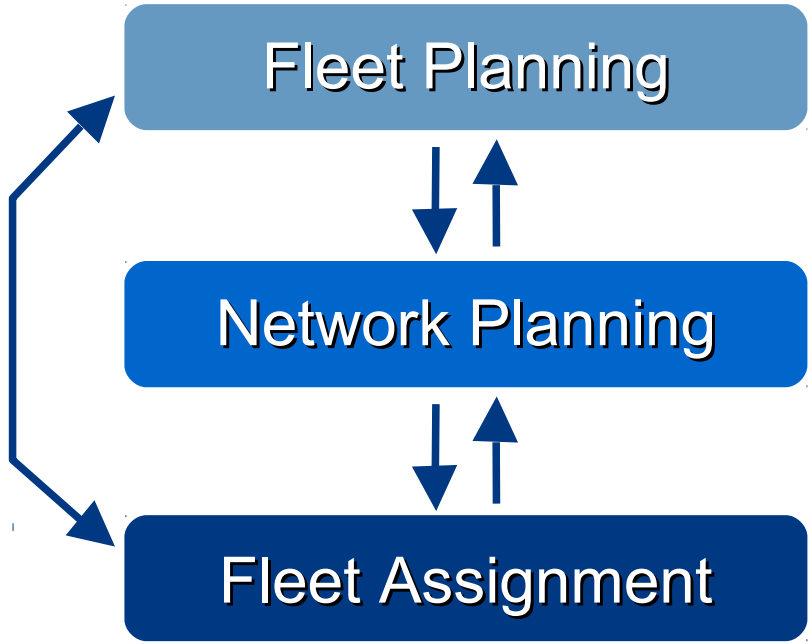
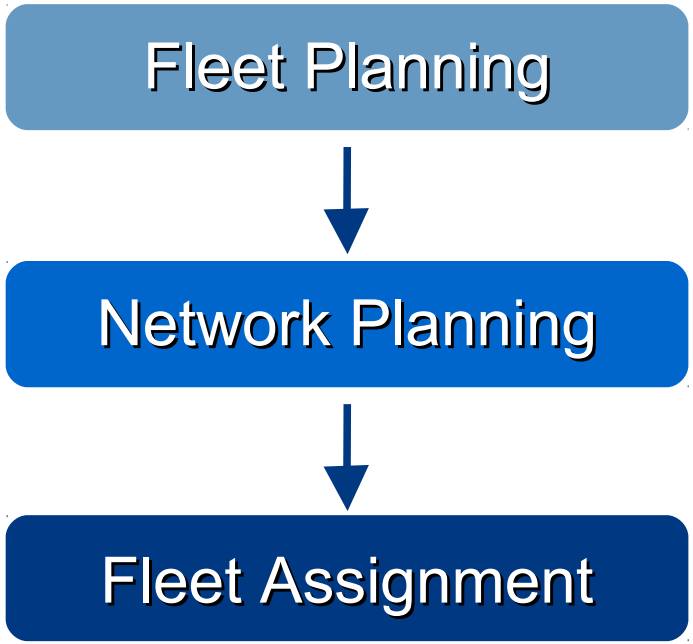


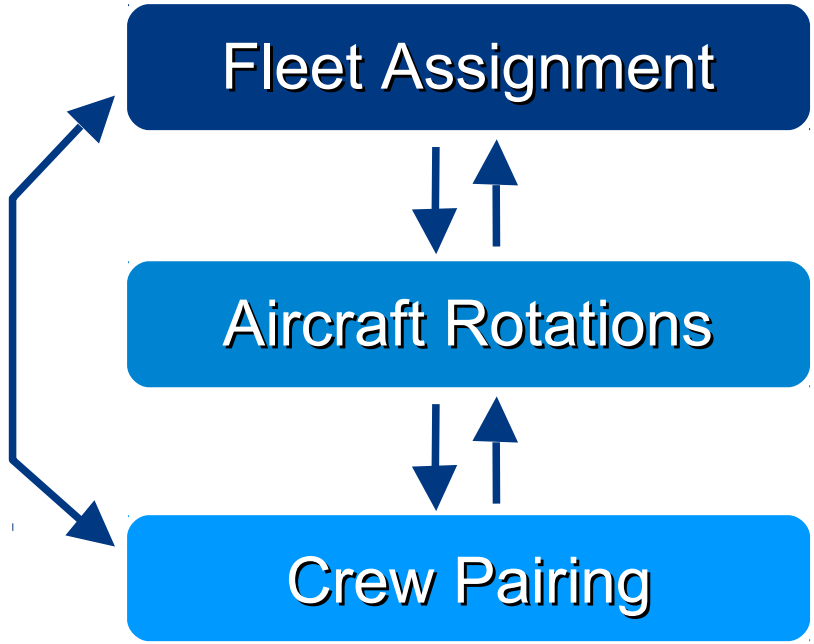
PlanUnified
unified optimization



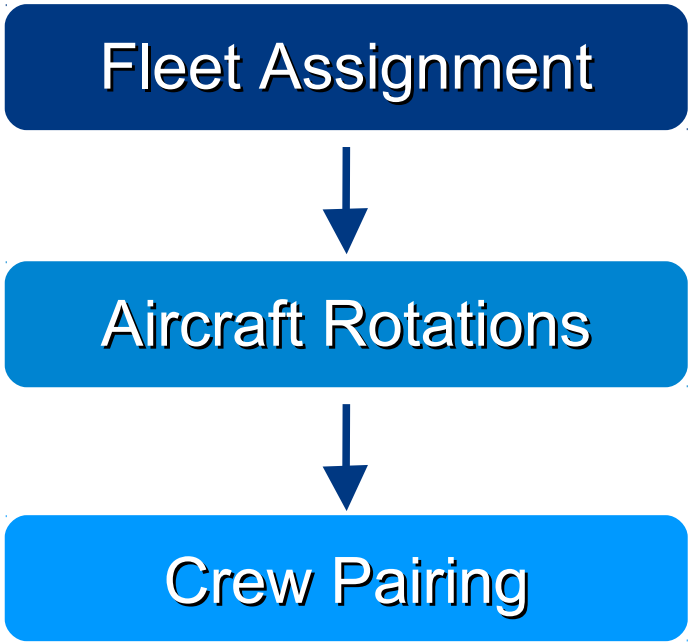
Other Products
sequential optimization



SchedUnified
unified optimization



Other Products
sequential optimization



Users report savings of

USD 300 / flight

within a month of using our system

$$\min \mathbf{c}^{\mathrm{T}} \cdot \boldsymbol{x} + \mathrm{c}(\boldsymbol{y})$$

$$\mathbf{A} \cdot \boldsymbol{x} + \boldsymbol{y} = \mathbf{b}$$

$$\boldsymbol{x} \geq \mathbf{0}, \boldsymbol{y} \in \mathbf{Y}$$

$$\min \mathbf{c}^T \cdot \boldsymbol{x} + c(\boldsymbol{y})$$
$$\mathbf{A} \cdot \boldsymbol{x} + \boldsymbol{y} = \mathbf{b}$$
$$\boldsymbol{x} \geq \mathbf{0},\ \boldsymbol{y} \in \mathbf{Y}$$

Master Problem

$$\min\ c(\boldsymbol{y}) + ?$$
$$???$$
$$\boldsymbol{y} \in \mathbf{Y}$$

Subproblem

$$\min\ \mathbf{c}^T \cdot \boldsymbol{x}$$
$$\mathbf{A} \cdot \boldsymbol{x} = \mathbf{b} - \tilde{\boldsymbol{y}}$$
$$\boldsymbol{x} \geq \mathbf{0}$$

Benders subproblem given a MP solution

$$\min \mathbf{c}^T \cdot \boldsymbol{x} + c(\boldsymbol{y})$$
$$\mathbf{A} \cdot \boldsymbol{x} + \boldsymbol{y} = \mathbf{b}$$
$$\boldsymbol{x} \geq \mathbf{0},\ \boldsymbol{y} \in \mathbf{Y}$$

Master Problem

$$\min\ c(\boldsymbol{y}) + ?$$
$$???$$
$$\boldsymbol{y} \in \mathbf{Y}$$

$\tilde{\boldsymbol{y}}$

Subproblem

$$\min\ \mathbf{c}^T \cdot \boldsymbol{x}$$
$$\mathbf{A} \cdot \boldsymbol{x} = \mathbf{b} - \tilde{\boldsymbol{y}}$$
$$\boldsymbol{x} \geq \mathbf{0}$$

Dual Benders subproblem

$$\min \mathbf{c}^T \cdot \boldsymbol{x} + c(\boldsymbol{y})$$
$$\mathbf{A} \cdot \boldsymbol{x} + \boldsymbol{y} = \mathbf{b}$$
$$\boldsymbol{x} \geq \mathbf{0},\ \boldsymbol{y} \in \mathbf{Y}$$

Master Problem

$$\min\ c(\boldsymbol{y}) + ?$$
$$???$$
$$\boldsymbol{y} \in \mathbf{Y}$$

Subproblem

$$\min\ \mathbf{c}^T \cdot \boldsymbol{x}$$
$$\mathbf{A} \cdot \boldsymbol{x} = \mathbf{b} - \tilde{\boldsymbol{y}}$$
$$\boldsymbol{x} \geq \mathbf{0}$$

↓

Dual of Subproblem

$$\max\ (\mathbf{b} - \tilde{\boldsymbol{y}})^T \cdot \boldsymbol{\pi}$$
$$\mathbf{A}^T \cdot \boldsymbol{\pi} \leq \mathbf{c}$$

Duality theorem

$$\min \mathbf{c}^T \cdot \boldsymbol{x} + c(\boldsymbol{y})$$
$$\mathbf{A} \cdot \boldsymbol{x} + \boldsymbol{y} = \mathbf{b}$$
$$\boldsymbol{x} \geq \mathbf{0},\ \boldsymbol{y} \in \mathbf{Y}$$

Master Problem

$\min\ c(\boldsymbol{y}) + ?$

???

$\boldsymbol{y} \in \mathbf{Y}$

Duality Theorem

$(\mathbf{b} - \tilde{\boldsymbol{y}})^T \cdot \boldsymbol{\pi} \leq z$

Subproblem

$z = \min\ \mathbf{c}^T \cdot \boldsymbol{x}$

$\mathbf{A} \cdot \boldsymbol{x} = \mathbf{b} - \tilde{\boldsymbol{y}}$

$\boldsymbol{x} \geq \mathbf{0}$

Dual of Subproblem

$\max\ (\mathbf{b} - \tilde{\boldsymbol{y}})^T \cdot \boldsymbol{\pi}$

$\mathbf{A}^T \cdot \boldsymbol{\pi} \leq \mathbf{c}$

Duality theorem

$$\min \mathbf{c}^T \cdot \boldsymbol{x} + c(\boldsymbol{y})$$
$$\mathbf{A} \cdot \boldsymbol{x} + \boldsymbol{y} = \mathbf{b}$$
$$\boldsymbol{x} \geq \mathbf{0},\ \boldsymbol{y} \in \mathbf{Y}$$

Master Problem

$\min\ c(\boldsymbol{y}) + ?$

$???$

$\boldsymbol{y} \in \mathbf{Y}$

$(\mathbf{b} - \tilde{\boldsymbol{y}})^T \cdot \boldsymbol{\pi} \leq z$

Subproblem

$z = \min\ \mathbf{c}^T \cdot \boldsymbol{x}$

$\mathbf{A} \cdot \boldsymbol{x} = \mathbf{b} - \tilde{\boldsymbol{y}}$

$\boldsymbol{x} \geq \mathbf{0}$

Dual of Subproblem

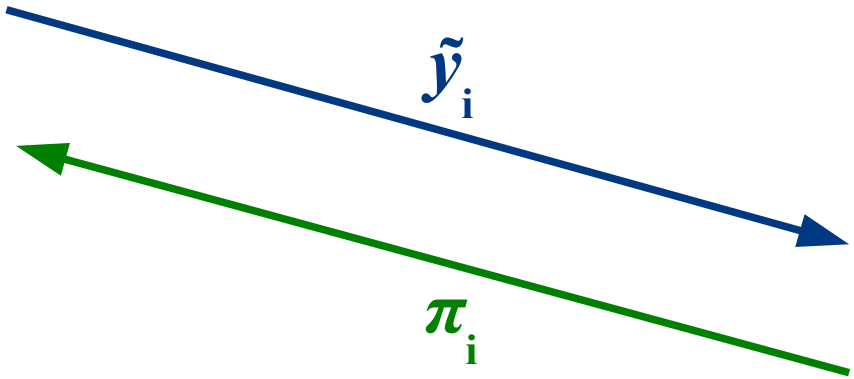
$\max\ (\mathbf{b} - \tilde{\boldsymbol{y}})^T \cdot \boldsymbol{\pi}$

$\mathbf{A}^T \cdot \boldsymbol{\pi} \leq \mathbf{c}$

$$\min \mathbf{c}^T \cdot \boldsymbol{x} + c(\boldsymbol{y})$$
$$\mathbf{A} \cdot \boldsymbol{x} + \boldsymbol{y} = \mathbf{b}$$
$$\boldsymbol{x} \geq 0, \boldsymbol{y} \in \mathbf{Y}$$

Master Problem

$$\min \; c(\boldsymbol{y}) + z$$
$$\ldots$$
$$(\mathbf{b} - \boldsymbol{y})^T \cdot \boldsymbol{\pi}_i \leq z$$
$$\ldots$$
$$\boldsymbol{y} \in \mathbf{Y}$$



Subproblem

$$\min \; \mathbf{c}^T \cdot \boldsymbol{x}$$
$$\mathbf{A} \cdot \boldsymbol{x} = \mathbf{b} - \tilde{\boldsymbol{y}}_i$$
$$\boldsymbol{x} \geq 0$$

Dual of Subproblem

$$\max \; (\mathbf{b} - \tilde{\boldsymbol{y}}_i)^T \cdot \boldsymbol{\pi}_i$$
$$\mathbf{A}^T \cdot \boldsymbol{\pi}_i \leq \mathbf{c}$$

decisal

making decisions optimal

- Born from cutting edge research at Imperial College London
- Based in London, UK
- IBM partner


IBM
IBM Global Entrepreneur Success Stories UK & Ireland 2013
Looking for Global Entrepreneurs who want to build a Smarter Planet


Business Analytics
VoiceSecure
developed a solution for disasters – from fires in apartment blocks to nuclear attacks – and a voice biometrics solution to help identify callers to customer care and help desks
ContruQtive
developed a flexible, scalable mobile and web-based reporting solution within the small business space that provides customised business insights and projections
Hiwayman
enabled a UK address to be captured quickly and accurately, and developed a solution to ensure that an account number is valid for a given sort code
Decisal
provided a new system that allows unified optimisation of air network aircraft maintenance rotation, as well as crew and fleet assignment planning

Conclusions

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